Lecture 4: Indefinites and Dependence

Team Semantics: Linguistic and Philosophical Applications

Aleksi Anttila Marco Degano

ESSLLI 2026, Prague

13 August 2026

Readings

- Degano, M., & Aloni, M. (2025). How to be (non-)specific? *Linguistics and Philosophy*, *48*, 955–1003. https://doi.org/10.1007/s10988-025-09442-y
- Brasoveanu, A., & Farkas, D. F. (2011). How indefinites choose their scope. *Linguistics and Philosophy*, *34*, 1–55. https://doi.org/10.1007/s10988-011-9092-7

Recall: strict vs. lax disjunction

$$M, T \models \phi \vee \psi$$

T	strict split		lax split	
i_1	•		•	
i_2	•		•	•
i_3		•		•
i_4		•	•	•

- **Strict**: every assignment goes to exactly one subteam.
- **Lax**: an assignment may go to both subteams.

Existential choice as generalized disjunction

Think of an existential as a big disjunction over witnesses:

$$\exists x\, \phi(x) \quad \rightsquigarrow \quad \bigvee_{d \in D} \phi(d)$$

For example, if $D = \{a, b, c\}$:

$$\exists x\, \phi(x) \quad \rightsquigarrow \quad \phi(a) \vee \phi(b) \vee \phi(c)$$

T	$\phi(a)$	$\phi(b)$	$\phi(c)$
i_1	●		
i_2		●	
i_3			●
i_4		●	

The existential distributes assignments over possible witnesses.

Strict vs. lax existential extension

$\exists x\, \phi \quad \rightsquigarrow \quad$ distribute the assignments in T over the possible values of x

Input team

T	y	z
i_1	0	0
i_2	0	1
i_3	1	1

$D = \{a, b, c\}$

0, 1
placeholders

Strict choice: a partition

	T_a	T_b	T_c
i_1	•		
i_2		•	
i_3			•

$T = T_a \dot\cup T_b \dot\cup T_c$

T'	y	z	x
$i_1[a/x]$	0	0	a
$i_2[b/x]$	0	1	b
$i_3[c/x]$	1	1	c

$F : T \to D$

$T[F/x] = \{i[F(i)/x] : i \in T\}$

Lax choice: an overlapping cover

	T_a	T_b	T_c
i_1	•		
i_2	•	•	
i_3		•	•

$T = T_a \cup T_b \cup T_c$

T'	y	z	x
$i_1[a/x]$	0	0	a
$i_2[a/x]$	0	1	a
$i_2[b/x]$	0	1	b
$i_3[b/x]$	1	1	b
$i_3[c/x]$	1	1	c

$H : T \to \mathcal{P}(D) \setminus \{\varnothing\}$

$T[H/x] = \{i[d/x] : i \in T,\ d \in H(i)\}$

Outline

A wealth of Indefinites

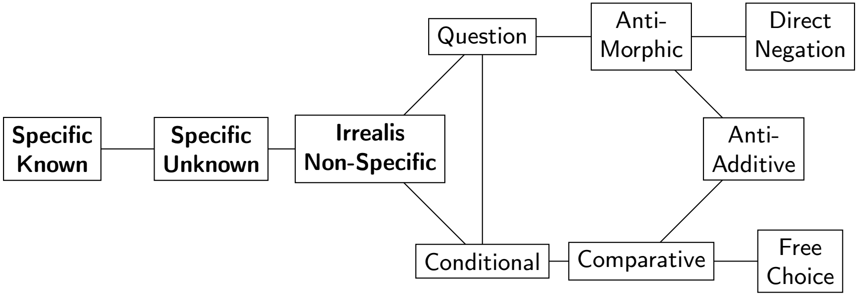
Cross-linguistically, we witness a wealth of indefinite forms:

- English: *some*, *any*, *no*, ...
- Italian: *qualcuno*, *qualunque*, *nessuno*, *(un) qualche*, ...
- Dutch: *iets*, *enig*, *wie dan ook*, *niets*, ...
- German: *ein*, *irgendein*, ...
- Russian: *koe-*, *-to*, *-nibud*, ...
- Spanish: *algún*, *cualquiera*, *ningun*, ...
- Czech: *někdo*, *nějaký*, *kdokoli*, *nikdo*, ...
- ...

Why this variety? What do all these forms have in common? How to account for their differences in meaning and distribution?

Haspelmath Map

Haspelmath (1997)'s map: a useful typological tool to capture the functional distribution of indefinites:



Haspelmath's map

- Indefinites need to occupy connected areas.
- Diachronic extensions tend to first add adjacent functions.

Scopal vs. epistemic specificity

Scopal specificity

(1) a. Ali wants to visit *an Italian city*.

b. <u>Specific [$\exists x/\Box$]</u>: there is a city x such that Ali wants to visit x.

c. <u>Non-specific [$\Box/\exists x$]</u>: Ali wants to visit an Italian city, any Italian city would do.

Epistemic specificity:

(2) a. *A student* called.

b. <u>Known:</u> the speaker knows which student called.

c. <u>Unknown:</u> the speaker does not know which student called.

Specific Known, Specific Unknown and Non-Specific

Reading	Scopal status	Epistemic status
Specific known (SK)	specific	known
Specific unknown (SU)	specific	unknown
Non-specific (NS)	non-specific	–

(3) Ali wants to visit an *Italian city*.

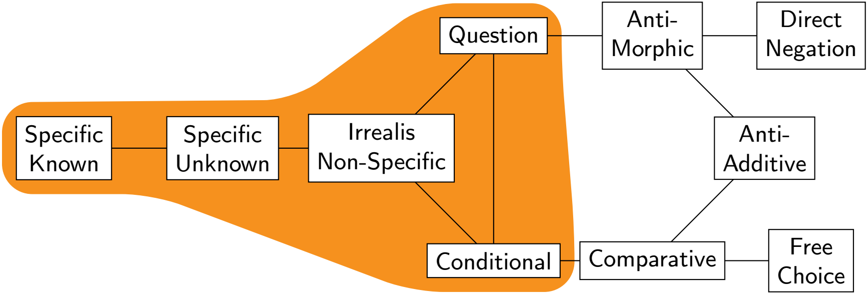
a. **SK:** There is a specific city which Ali wants to visit, and the speaker knows which

b. **SU:** There is a specific city which Ali wants to visit, but the speaker does not know which

c. **NS:** Ali wants to visit an Italian city, any Italian city would do

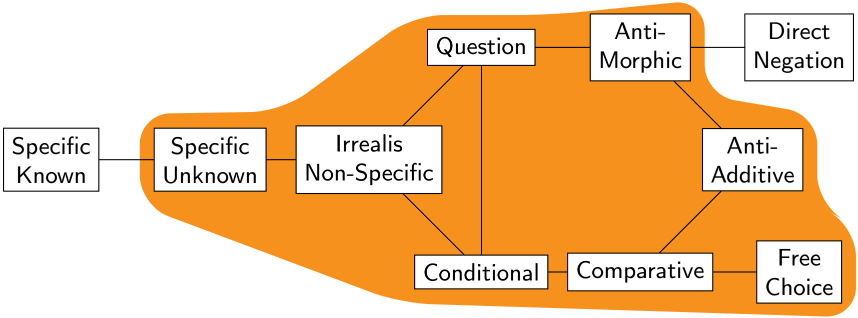
Cross-linguistically, languages develop lexicalized forms with restricted distributions with respect to these uses.

Haspelmath Map



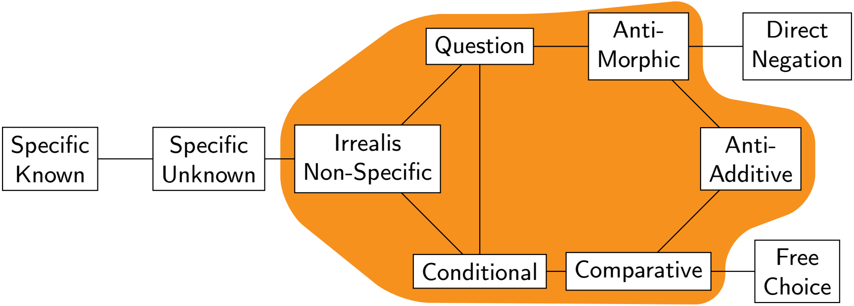
English *someone*

Haspelmath Map



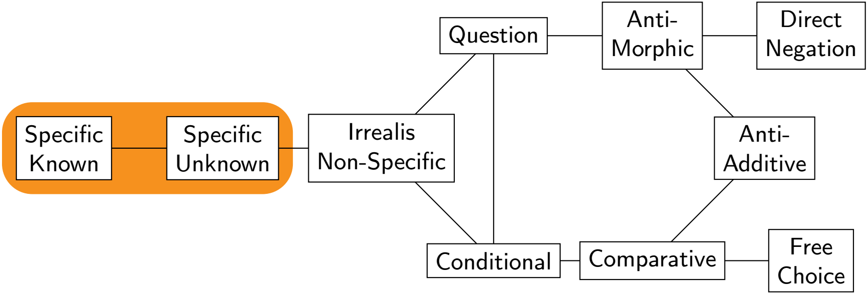
German *irgend-*

Haspelmath Map



Russian *nibud'*

Haspelmath Map



Kazakh *älde*

Our Goals

(1) the logical characterization of the specific known (SK), specific unknown (SU) and non-specific (NS) uses;

(2) a formal account of the variety of marked indefinites encoding SK, SU, and NS; and their properties.

Main idea: Indefinites are sensitive to dependence and non-dependence relationships in their value assignments. (building on insights from Brasoveanu and Farkas 2011; Farkas and Brasoveanu 2020).

Implementation: Two-sorted team semantics with dependence atoms.

Marked Indefinites

Possible **marked indefinites** based on Specific Known (SK), Specific Unknown (SU) and Non-specific (NS):

TYPE	FUNCTIONS SK	SU	NS	EXAMPLE
(i) unmarked	✓	✓	✓	Italian *qualcuno*
(ii) specific	✓	✓	×	Georgian *-ghats*
(iii) non-specific	×	×	✓	Russian *-nibud*
(iv) epistemic	×	✓	✓	German *irgend-*
(v) specific known	✓	×	×	Russian *koe-*
(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

Marked Indefinites

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(iii) non-specific	×	×	✓	Russian *-nibud*
(iv) epistemic	×	✓	✓	German *irgend-*
(v) specific known	✓	×	×	Russian *koe-*
(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

How to capture this variety?

Marked Indefinites

Possible **marked indefinites** based on Specific Known (SK), Specific Unknown (SU) and Non-specific (NS):

TYPE	FUNCTIONS SK	FUNCTIONS SU	FUNCTIONS NS	EXAMPLE
(i) unmarked	✓	✓	✓	Italian *qualcuno*
(ii) specific	✓	✓	×	Georgian *-ghats*
(iii) non-specific	×	×	✓	Russian *-nibud*
(iv) epistemic	×	✓	✓	German *irgend-*
(v) specific known	✓	×	×	Russian *koe-*
(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

Why non-specific have a restricted distribution (ungrammatical in episodic contexts)?

Marked Indefinites

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TYPE	FUNCTIONS			EXAMPLE
	SK	SU	NS	
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(iii) non-specific	×	×	✓	Russian *-nibud*
(iv) epistemic	×	✓	✓	German *irgend-*
(v) specific known	✓	×	×	Russian *koe-*
(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

How to characterize the obligatory ignorance inferences typical of epistemic indefinites?

Marked Indefinites

Possible **marked indefinites** based on Specific Known (SK), Specific Unknown (SU) and Non-specific (NS):

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(iii) non-specific	×	×	✓	Russian *-nibud*
(iv) epistemic	×	✓	✓	German *irgend-*
(v) specific known	✓	×	×	Russian *koe-*
(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

Why diachronically non-specific indefinites tend to turn into epistemic ones?

Marked Indefinites

Possible **marked indefinites** based on Specific Known (SK), Specific Unknown (SU) and Non-specific (NS):

TYPE	FUNCTIONS SK	SU	NS	EXAMPLE
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(v) specific known	✓	×	×	Russian *koe-*
(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

Why (vi) is unattested and (vii) rare?

Marked Indefinites

Possible **marked indefinites** based on Specific Known (SK), Specific Unknown (SU) and Non-specific (NS):

TYPE	FUNCTIONS SK	SU	NS	EXAMPLE
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(vi) SK + NS	✓	×	✓	unattested
(vii) specific unknown	×	✓	×	Kannada *-oo*

Indefinites in general display exceptional scope behaviour. Why? How to account for their exceptional scope? What scope configurations are possible for marked indefinites (e.g. narrow, intermediate, wide)?

Outline

Two-sorted language and models

A model is a triple $M = \langle D, W, I \rangle$:

- D: domain of individuals;
- W: domain of possible worlds;
- I: interpretation function, allowing intensional predicates $P(\vec{x}, w)$.

Variables are sorted:

- individual variables: $x, y, z, \ldots$;
- world variables: $v, w, u, \ldots$;
- v is a distinguished variable: it represents the speaker's epistemic possibilities.

A team is a set of assignments with a fixed finite domain. Each assignment maps individual variables to D and world variables to W.

Language

$$\begin{aligned}\phi ::= P(\vec{z}) \mid \neg P(\vec{z}) \mid \phi \wedge \psi \mid \phi \vee \psi \\ \mid \forall z\, \phi \mid \exists_{\mathrm{s}} z\, \phi \mid \exists_{\mathrm{l}} z\, \phi \\ \mid \mathsf{dep}(\vec{z}, u) \mid \mathsf{var}(\vec{z}, u)\end{aligned}$$

Here z, u range over variables of either sort, but predicate arguments are type-correct (individual vs world variables)

Convention: In examples, $\phi(x, w)$ abbreviates the nuclear scope containing the relevant predicate at world w. For simple episodic examples, $\phi(x, v)$ is evaluated at the epistemic world variable v.

Teams as information states: initial teams

Teams represent information states of speakers.

In initial teams only factual information is represented.

Initial team: A nonempty team T is *initial* iff $\mathrm{Dom}(T) = \{v\}$.

- The *designated world variable* v captures the speaker's epistemic possibilities.
- Teams where v receives only one value are teams of *maximal information*.

Felicitous sentence: A sentence is *felicitous/grammatical* if there is an initial team which supports it.

Initial teams

v	v
w_{Pa}	w_{Pa}
w_{Pb}	
initial team	initial team of maximal information

- The team on the left conveys the info that Pa is true or Pb is true.
- The team on the right conveys the infor that Pa is true. A team of maximal information.
- w_{Pa} means here $\langle I_M(a), w_{Pa}\rangle \in I_M(P)$.

Universal Extension

$$T[y] = \{i[d/y] : i \in T \text{ and } d \in D\}$$

A **universal extension** of a team T with y, denoted by $T[y]$, amounts to consider all assignments that differ from the ones in T only with respect to the value of y.

v	T
v_1	i_1
v_2	i_2

v	y	$T[y]$
v_1	d_1	i_{11}
	d_2	i_{12}
v_2	d_1	i_{21}
	d_2	i_{22}

($D = \{d_1, d_2\}$. Universal extensions are unique.)

Strict Functional Extension

$$T[h/y] = \{i[h(i)/y] : i \in T\}, \text{for some function } h : T \to D$$

A **strict functional extension** of a team T with y, $T[h/y]$, amounts to assign only one value to y for each original assignment in T.

v	T
v_1	i_1
v_2	i_2

With $D = \{d_1, d_2\}$ we have 4 possible strict functional extensions:

v	y	$T[h_1/y]$
v_1	$\to d_1$	i_{12}
v_2	$\to d_1$	i_{21}

v	y	$T[h_2/y]$
v_1	$\to d_2$	i_{12}
v_2	$\to d_2$	i_{21}

x	y	$T[h_3/y]$
v_1	$\to d_1$	i_{12}
v_2	$\to d_2$	i_{21}

x	y	$T[h_4/y]$
v_1	$\to d_2$	i_{12}
v_2	$\to d_1$	i_{21}

Lax Functional Extension

$T[f/y] = \{i[d/y] : i \in T \text{ and } d \in f(i)\}$, for some function $f : T \to \wp(D) \setminus \{\varnothing\}$

A **lax functional extension** of a team T with y, $T[h/y]$, amounts to assign one or more values to y for each original assignment in T.

v	T
v_1	i_1
v_2	i_2

v	y	$T[f/y]$
$v_1 \to$	d_2	i_{12}
$v_2 \to$	d_1	i_{21}
$v_2 \to$	d_2	i_{22}

(With $D = \{d_1, d_2\}$ we have 9 possible lax functional extensions. Branching allowed)

Semantic Clauses

$$
\begin{array}{lcl}
M, T \models P(x_1, \ldots, x_n) & \Leftrightarrow & \forall j \in T : \langle j(x_1), \ldots, j(x_n) \rangle \in I(P^n) \\
M, T \models \phi \wedge \psi & \Leftrightarrow & M, T \models \phi \text{ and } M, T \models \psi \\
M, T \models \phi \vee \psi & \Leftrightarrow & T = T_1 \cup T_2 \text{ for teams } T_1 \text{ and } T_2 \text{ s.t. } M, T_1 \models \phi \text{ and } M, T_2 \models \psi \\
M, T \models \forall y \phi & \Leftrightarrow & M, T[y] \models \phi, \text{ where } T[y] = \{i[d/y] : i \in T \text{ and } d \in D\} \\
M, T \models \exists_{\text{strict}} y \phi & \Leftrightarrow & \text{there is a function } h : T \rightarrow D \text{ s.t. } M, T[h/y] \models \phi, \text{ where } T[h/y] = \{i[h(i)/y] : i \in T\} \\
M, T \models \exists_{\text{lax}} y \phi & \Leftrightarrow & \text{there is a function } f : T \rightarrow \wp(D) \backslash \{\varnothing\} \text{ s.t. } M, T[f/y] \models \phi, \text{ where } T[f/y] = \{i[d/y] : i \in T \text{ and } d \in f(i)\} \\
M, T \models dep(\vec{x}, y) & \Leftrightarrow & \text{for all } i, j \in T : i(\vec{x}) = j(\vec{x}) \Rightarrow i(y) = j(y) \\
M, T \models var(\vec{x}, y) & \Leftrightarrow & \text{there is } i, j \in T : i(\vec{x}) = j(\vec{x}) \& i(y) \neq j(y)
\end{array}
$$

Dependence Atoms

Dependence atoms (Väänänen 2007; Galliani 2015) model dependency patterns between variables' values:

Dependence Atom:

$$M, T \models dep(\vec{x}, y) \Leftrightarrow \text{ for all } i, j \in T : i(\vec{x}) = j(\vec{x}) \Rightarrow i(y) = j(y)$$

Variation Atom:

$$M, T \models var(\vec{x}, y) \Leftrightarrow \text{ there is } i, j \in T : i(\vec{x}) = j(\vec{x}) \& i(y) \neq j(y)$$

T	x	y	z	l
i	a_1	b_1	c_1	d_1
j	a_1	b_1	c_2	d_1
k	a_3	b_2	c_3	d_1

$dep(x, y)$ ✓ $\quad$ $var(x, z)$ ✓

$dep(\varnothing, l)$ ✓ $\quad$ $var(\varnothing, x)$ ✓

$dep(xy, z)$ $\times$ $\quad$ $var(x, y)$ $\times$

Outline

Indefinites as Existentials

We propose that:

1. Indefinites are **strict existentials** ($\exists_{\mathsf{s(trict)}} x$).
2. They are interpreted *in-situ*.

Dependence atoms can be used to model the **scope** behaviour of indefinites, by specifying how their value (co-)varies with other operators.

(For scope, our system parallels Brasoveanu and Farkas (2011)'s treatment).

Application I: Exceptional Scope

Indefinites violate rules of standard quantifier behaviour, e.g, can escape syntactic islands (Reinhart 1979, Abush 1993, . . .)

(4) Every kid$_x$ ate every food$_z$ that a doctor$_y$ recommended.

a. WS $[\exists y/\forall x/\forall z]$: $\forall x \forall z \exists_s y(\phi \wedge dep(v, y))$

b. IS $[\forall x/\exists y/\forall z]$: $\forall x \forall z \exists_s y(\phi \wedge dep(vx, y))$

c. NS $[\forall x/\forall z/\exists y]$: $\forall x \forall z \exists_s y(\phi \wedge dep(vxz, y))$

v	x	z	y
v_1	...	...	b_1
v_1	...	...	b_1
v_1	...	...	b_1
v_1	...	...	b_1

WS: $dep(v, y)$

v	x	z	y
v_1	a_1	...	b_1
v_1	a_1	...	b_1
v_1	a_2	...	b_2
v_1	a_2	...	b_2

IS: $dep(vx, y)$

v	x	z	y
v_1	a_1	c_1	b_1
v_1	a_1	c_2	b_2
v_1	a_2	c_3	b_3
v_1	a_2	c_4	b_4

NS: $dep(vxz, y)$

Indefinites interpreted *in-situ*. Exceptional scope behaviour captured using dependence atoms.

Application II: Specific Known, Specific Unknown, Non-specific

constancy	$dep(\varnothing, x)$	$\begin{array}{cc} v & x \\ \hline \ldots & d_1 \\ \hline \ldots & d_1 \end{array}$	v-constancy	$dep(v, x)$	$\begin{array}{cc} v & x \\ \hline v_1 & d_1 \\ \hline v_2 & d_2 \end{array}$
variation	$var(\varnothing, x)$	$\begin{array}{cc} v & x \\ \hline \ldots & d_1 \\ \hline \ldots & d_2 \end{array}$	v-variation	$var(v, x)$	$\begin{array}{cc} v & x \\ \hline v_1 & d_1 \\ \hline v_1 & d_2 \end{array}$

Specific Known:
constancy $dep(\varnothing, x)$

$$\begin{array}{ccc} v & \ldots & x \\ \hline v_1 & \ldots & d_1 \\ \hline v_2 & \ldots & d_1 \end{array}$$

Application II: Specific Known, Specific Unknown, Non-specific

constancy	$dep(\varnothing, x)$	v	x		v-constancy	$dep(v, x)$	v	x	
		$\ldots$	d_1				v_1	d_1	
		$\ldots$	d_1				v_2	d_2	
variation	$var(\varnothing, x)$	v	x		v-variation	$var(v, x)$	v	x	
		$\ldots$	d_1				v_1	d_1	
		$\ldots$	d_2				v_1	d_2	

Specific Unknown:
v-constancy $dep(v, x)$ $+$
variation $var(\varnothing, x)$

v	$\ldots$	x
v_1	$\ldots$	d_1
v_2	$\ldots$	d_2

Application II: Specific Known, Specific Unknown, Non-specific

constancy	$dep(\varnothing, x)$	$\begin{array}{cc} v & x \\ \hline \ldots & d_1 \\ \hline \ldots & d_1 \end{array}$	v-constancy	$dep(v, x)$	$\begin{array}{cc} v & x \\ \hline v_1 & d_1 \\ \hline v_2 & d_2 \end{array}$
variation	$var(\varnothing, x)$	$\begin{array}{cc} v & x \\ \hline \ldots & d_1 \\ \hline \ldots & d_2 \end{array}$	v-variation	$var(v, x)$	$\begin{array}{cc} v & x \\ \hline v_1 & d_1 \\ \hline v_1 & d_2 \end{array}$

Non-specific:
v-variation $var(v, x)$

$$\begin{array}{ccc} v & \ldots & x \\ \hline v_1 & \ldots & d_1 \\ \hline v_1 & \ldots & d_2 \end{array}$$

Application II: Specific Known, Specific Unknown, Non-specific

reading	**required pattern**	**scope**	**epistemic status**
SK	$\text{dep}(\varnothing, x)$	one witness across worlds	speaker-resolved
SU	$\text{dep}(v, x) \wedge \text{var}(\varnothing, x)$	one witness per world	speaker-unresolved
NS	$\text{var}(v, x)$	alternatives within a world	not referentially fixed

SK

v	x
v_1	a
v_2	a

SU

v	x
v_1	a
v_2	b

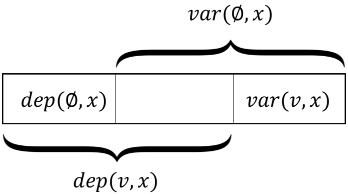
NS

v	y	x
v_1	c	a
v_1	d	b

Application III: Variety of Indefinites

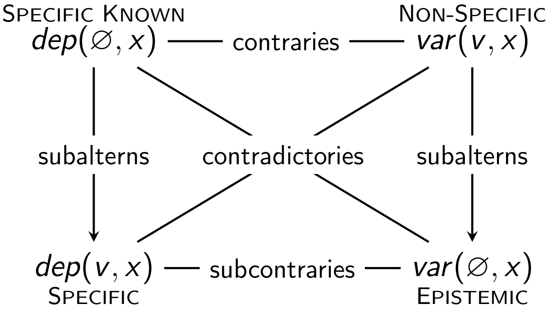
TYPE	FUNCTIONS			REQUIREMENT	EXAMPLE
	SK	SU	NS		
(i) unmarked	✓	✓	✓	none	Italian *qualcuno*
(ii) specific	✓	✓	×	$dep(v, x)$	Georgian *-ghats*
(iii) non-specific	×	×	✓	$var(v, x)$	Russian *-nibud*
(iv) epistemic	×	✓	✓	$var(\varnothing, x)$	German *-irgend*
(v) specific known	✓	×	×	$dep(\varnothing, x)$	Russian *-koe*
(vi) SK + NS	✓	×	✓	$dep(\varnothing, x) \mathbin{\mathbb{V}} var(v, x)$	unattested
(vii) specific unknown	×	✓	×	$dep(v, x) \wedge var(\varnothing, x)$	Kannada *-oo*

(vi) SK + NS: violation of connectedness/convexity



(vii) specific unknown: increased complexity

Dependence Square of Opposition



Dependence Square of Opposition

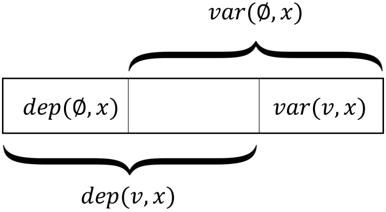
- Contraries: can be both false, but not both true.
- Contradictories: cannot be both true and they cannot be both false.
- Subcontraries: they cannot both be false but can both be true.
- Subalternation: A subalternates B iff A implies B.

Convexity

Convexity is a constraint on lexicalization: meanings tend to form convex regions in an appropriate meaning space.

It has been used for several semantic domains:

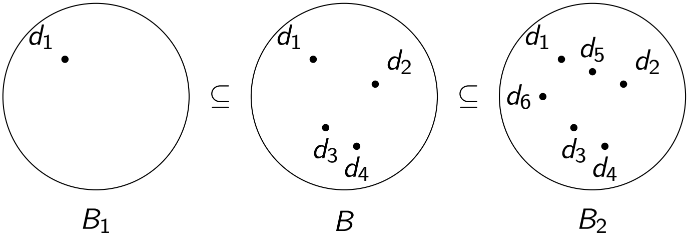
- colour terms and conceptual spaces (Gärdenfors 2000; Gärdenfors 2014; Jäger 2007; Jäger 2010);
- generalized quantifiers (van Benthem 1984; Steinert-Threlkeld and Szymanik 2020);
- modals (Steinert-Threlkeld, Imel, et al. 2023).



Convexity in Generalized Quantifiers

A determiner Q is convex in its nuclear-scope argument iff, for all M and all $A, B_1, B, B_2 \subseteq M$,

$$B_1 \subseteq B \subseteq B_2 \quad \text{and} \quad Q_M(A, B_1),\ Q_M(A, B_2) \quad \Rightarrow \quad Q_M(A, B)$$



- *some*, *no*, *less than two*, *more than two and less than five* are convex
- *less than two or more than five* is not convex

van Benthem (1984)

Convexity in team semantics

In team semantics, meanings are sets of teams:

$$[\![\phi]\!]_M = \{T : M, T \models \phi\}$$

A set of teams P is convex iff, for all teams $T \subseteq T' \subseteq T''$,

$$T \in P \;\&\; T'' \in P \quad \Rightarrow \quad T' \in P$$

For the unattested $\mathrm{SK{+}NS}$ type:

$$P_{\mathrm{SK+NS}} = [\![\mathsf{dep}(\varnothing, x)]\!]_M \cup [\![\mathsf{var}(v, x)]\!]_M$$

This property is not convex.

Why SK+NS is not convex

$$P_{\mathrm{SK+NS}} = [\![\mathsf{dep}(\varnothing, x)]\!]_M \cup [\![\mathsf{var}(v, x)]\!]_M$$

v	x
v_1	d_1

T

$\mathsf{dep}(\varnothing, x)$

v	x
v_1	d_1
v_2	d_2

T'

neither

v	x
v_1	d_1
v_2	d_2
v_1	d_2

T''

$\mathsf{var}(v, x)$

We have $T \subseteq T' \subseteq T''$ and $T, T'' \in P_{\mathrm{SK+NS}}$, but $T' \notin P_{\mathrm{SK+NS}}$.

Ordering on the Haspelmath map

The Boolean union of the SK and NS cells is not convex:

$$\mathrm{SK} + \mathrm{NS}: \qquad \mathsf{dep}(\varnothing, x) \veebar \mathsf{var}(v, x)$$

The two adjacent combinations are convex:

$$\mathrm{SK} + \mathrm{SU}: \qquad \mathsf{dep}(\varnothing, x) \veebar (\mathsf{var}(\varnothing, x) \wedge \mathsf{dep}(v, x)) \equiv \mathsf{dep}(v, x)$$

$$\mathrm{SU} + \mathrm{NS}: \qquad (\mathsf{var}(\varnothing, x) \wedge \mathsf{dep}(v, x)) \veebar \mathsf{var}(v, x) \equiv \mathsf{var}(\varnothing, x)$$

Contiguous cells on an implicational map should denote convex properties.

$\mathrm{SK} - \mathrm{SU} - \mathrm{NS}$	yes
$\mathrm{SU} - \mathrm{SK} - \mathrm{NS}$	no
$\mathrm{SK} - \mathrm{NS} - \mathrm{SU}$	no

Convexity: indefinites and quantifiers

Indefinite type	Requirement	Convex	Monotonic
Specific known	dep($\varnothing, x$)	$\checkmark$	$\downarrow$
Specific	dep(v, x)	$\checkmark$	$\downarrow$
Epistemic	var($\varnothing, x$)	$\checkmark$	$\uparrow$
Non-specific	var(v, x)	$\checkmark$	$\uparrow$
Specific unknown	dep(v, x) $\wedge$ var($\varnothing, x$)	$\checkmark$	$\times$
SK+NS	dep($\varnothing, x$) $\mathbin{\vee\!\!\!\vee}$ var(v, x)	$\times$	$\times$

Quantifier	Condition	Convex	Monotonic
no	$A \cap B = \varnothing$	$\checkmark$	$\downarrow$
less than 2	$\|A \cap B\| < 2$	$\checkmark$	$\downarrow$
some	$A \cap B \neq \varnothing$	$\checkmark$	$\uparrow$
every	$A \subseteq B$	$\checkmark$	$\uparrow$
more than 2 and less than 5	$2 < \|A \cap B\| < 5$	$\checkmark$	$\times$
less than 2 or more than 5	$\|A \cap B\| < 2$ or $\|A \cap B\| > 5$	$\times$	$\times$

Outline

Application IV: Licensing of non-specific indefinites

Non-specific indefinites are ungrammatical in episodic sentences and they need an operator (e.g. a universal quantifier or a modal) which licenses them:

(5)*	*Ivan*	*včera*	*kupil*	*kakuju-nibud'*	*knigu.*
	Ivan	yesterday	bought	which-INDEF.	book.

'Ivan bought some book [non-specific] yesterday.'

(6)	*Ivan*	***hotel***	*spet'*	*kakuju-nibud'*	*pesniu.*
	Ivan	want-PAST	sing-INF	which-INDEF.	song.

'Ivan wanted to sing some song [non-specific].'

Application IV: Licensing of non-specific indefinites

Recall that non-specific indefinites trigger v-variation: $var(v, x)$.

v
v_1

Application IV: Licensing of non-specific indefinites

Recall that non-specific indefinites trigger v-variation: $var(v, x)$.

$$\exists_s x\ (\phi \wedge var(v, x))$$

v
v_1

v	x
v_1	a_1

No initial team can support $\exists_s x(\varphi \wedge var(v, x))$

Application IV: Licensing of non-specific indefinites

Recall that non-specific indefinites trigger v-variation: $var(v, x)$.

$$\forall y\ \phi$$

v
v_1

v	y
v_1	b_1
	b_2

Application IV: Licensing of non-specific indefinites

Recall that non-specific indefinites trigger v-variation: $var(v, x)$.

$$\forall y \exists_s x \; (\phi \wedge var(v, x))$$

v
v_1

v	y
v_1	b_1
	b_2

v	y	x
v_1	b_1	a_1
	b_2	a_2

Modality: Licensing

Non-specific indefinites can also be licensed by modals:

(7)*	*On*	*kupil*	*kakoj-nibud'*	*tort.*	
	He	buy-PAST	some-nibud	cake.	

'He bought a cake.'

(8)	*On*	***mog***	*kupit'*	*kakoj-nibud'*	*tort.*
	He	can-PAST	buy-INF	some-nibud	cake

'He could buy a cake.

Modality: Flavours

We treat modals as **(lax) quantifiers** over worlds:
$\Box_w \sim \forall w$ and $\Diamond_w \sim \exists_{l(ax)} w$

(9) Necessity Modal

a. You must take some-*nibud* book

b. $\forall w \exists_s x(\phi(x, w) \wedge var(v, x))$

(10) Possibility Modal

a. You may take some-*nibud* book

b. $\exists_l w \exists_s x(\phi(x, w) \wedge var(v, x))$

Licensing: Illustration

We obtain the correct licensing behaviour.

$$\exists_l w \exists_s x(\phi(x, w) \wedge var(v, x))$$

v
v_1
v_2

v	w
v_1	w_1
	w_2
v_2	w_1

v	w	x
v_1	w_1	a_1
	w_2	a_2
v_2	w_1	a_1

Epistemic vs Deontic

(11) **Epistemic** vs **Deontic**

a. John might be in Paris.

b. John is allowed to go to Paris.

Only Epistemic modals give rise to **epistemic contradictions**:

(12) # John might be in Paris and he is not in Paris.

Epistemic modals: quantify over epistemic possibilities of the speaker (encoded by v in our system).

Deontic modals: interpreted wrt 'normative' rules, not necessarily compatible with the state of affairs in the actual world.

Inclusion Atoms

Indefinites: values of individual variables introduced by an indefinite
$\Rightarrow$ Dependence and Variation Atoms

Modals: values of world variables introduced by modals
$\Rightarrow$ Inclusion Atoms

Inclusion Atom:

$$M, T \models \vec{x} \subseteq \vec{y} \Leftrightarrow \text{ for all } i \in T, \text{ there is a } j \in T : i(\vec{x}) = j(\vec{y})$$

x	y	z
d_1	d_1	d_2
d_1	d_2	d_2
d_2	d_3	d_4
d_2	d_4	d_4

$x \subseteq y$ ✓
$xz \subseteq xy$ ✓
$y \subseteq x$ $\times$

Epistemic vs Deontic

(13) Epistemic

a. # John might be in Paris and he is not in Paris.

b. $\exists_1 w\,(P(j,w) \wedge w \subseteq v) \wedge \neg P(j,v) \models \bot$

(14) Deontic

a. John is allowed to be in Paris.

b. $\exists_1 w\,(P(j,w) \wedge R(v,w)) \wedge \neg P(j,v) \not\models \bot$

v
v_1
v_2
v_3

initial team

v	w
v_1	v_1
v_1	v_2
v_2	v_1
v_2	v_2
v_3	v_1
v_3	v_2

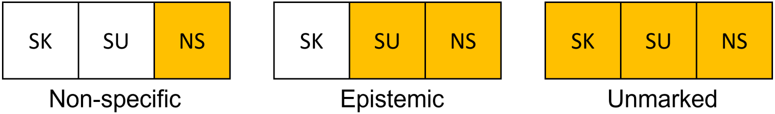
epistemic

v	w
v_1	w_1
v_1	w_2
v_2	w_1
v_2	w_2
v_3	w_1
v_3	w_2

deontic

Application V: Indefinites and Diachrony

Frequent diachronic tendency: non-specific > **epistemic** (e.g. French *quelque* (Foulet 1919) and German *irgendein* (Port and Aloni 2015))

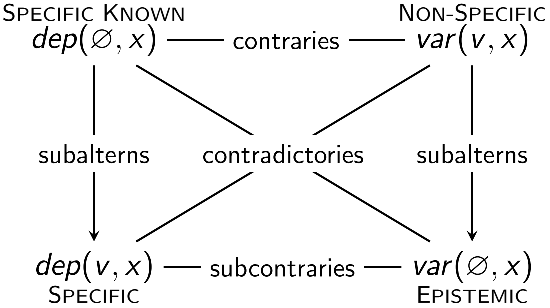


Haspelmath (1997)'s explanation: weakening of functions from the right (non-specific) of the functional map to the left (specific).

(15) Weakening of functions (a) > (b) > (c)
(a) non-specific
(b) non-specific + specific unknown = epistemic
(c) epistemic + specific known = unmarked

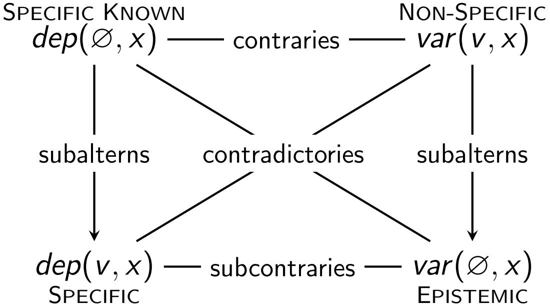
But then why diachronically we do not observe the change from (b) to (c)?

Indefinites and Diachrony



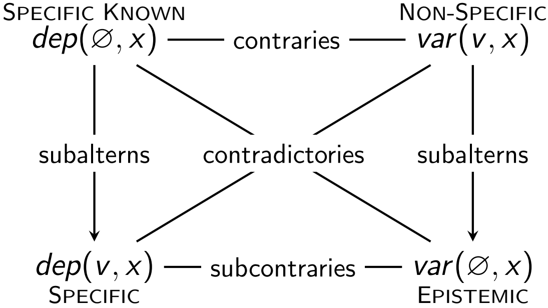
No changes between contradictories.

Indefinites and Diachrony



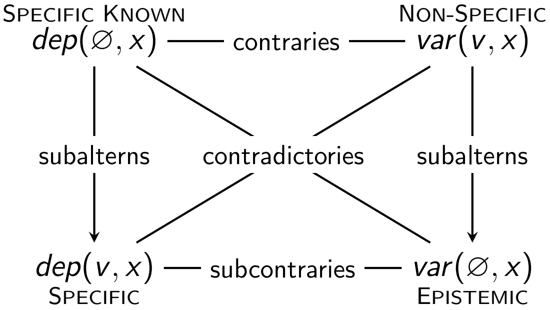
No changes between contraries (violation of convexity).

Indefinites and Diachrony



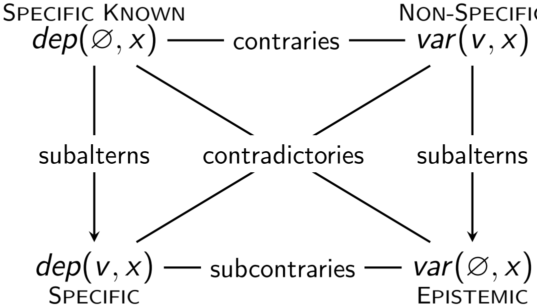
Changes between contraries possible, but the indefinite must both gain and lose a function.

Indefinites and Diachrony



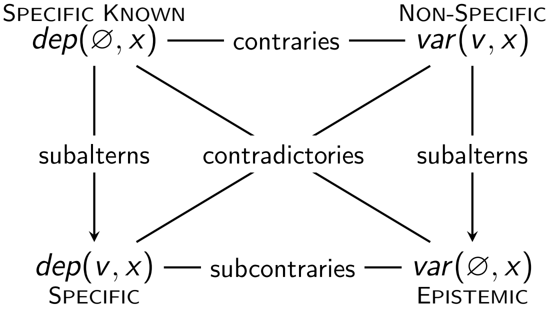
Changes between subalterns correspond to semantic weakening or strengthening (Heine 1997; Traugott and Dasher 2002; Hopper and Traugott 2003)

Indefinites and Diachrony



specific known	$\rightarrow$	unmarked	[+2 functions]
specific	$\rightarrow$	unmarked	[+1 function]
epistemic	$\rightarrow$	unmarked	[+1 function]
non-specific	$\rightarrow$	unmarked	[+2 functions]

Indefinites and Diachrony



Worlds are more abstract entities than individuals: from concrete to abstract concepts (Heine 1997; Port and Aloni 2021).

Language $\mathcal{L}_1$ without world variables:

$dep(\varnothing, x)_{\mathcal{L}_1}$: specific $\qquad$ $var(\varnothing, x)_{\mathcal{L}_1}$: non-specific

Language $\mathcal{L}_2$ with world variables:

$dep(\varnothing, x)_{\mathcal{L}_2}$: specific known $\qquad$ $var(\varnothing, x)_{\mathcal{L}_2}$: epistemic

$$\mathcal{L}_1 \rightarrow \mathcal{L}_2$$

Indefinites and Diachrony

Old Indefinite	New Indefinite	$\Rightarrow / \Leftarrow$	Functional Gain	Concreteness	Example
specific known $dep(\varnothing, x)$	specific $dep(v, x)$	$\Rightarrow$	+1	$\varnothing \rightarrow v$	unattested
specific known $dep(\varnothing, x)$	unmarked	$\Rightarrow$	+2	-	unattested
specific $dep(v, x)$	specific known $dep(\varnothing, x)$	$\Leftarrow$	-1	$v \rightarrow \varnothing$	unattested
specific $dep(v, x)$	epistemic $var(\varnothing, x)$	neither	-1;+1	$v \rightarrow \varnothing$	unattested
specific $dep(v, x)$	unmarked	$\Rightarrow$	+1	-	English *one*
epistemic $var(\varnothing, x)$	specific $dep(v, x)$	neither	-1;+1	$\varnothing \rightarrow v$	unattested
epistemic $var(\varnothing, x)$	non-specific $var(v, x)$	$\Leftarrow$	-1	$\varnothing \rightarrow v$	Italian *alcun(o)*
epistemic $var(\varnothing, x)$	unmarked	$\Rightarrow$	+1	-	Icelandic *nokkur*
non-specific $var(v, x)$	epistemic $var(\varnothing, x)$	$\Rightarrow$	+1	$v \rightarrow \varnothing$	German *irgend-*
non-specific $var(v, x)$	unmarked	$\Rightarrow$	+2	-	unattested

Final Proposal

We propose that:

1. Indefinites are **strict existentials**
2. They are interpreted **in-situ**
3. An unmarked/plain indefinite $\exists_s x$ in **syntactic scope** of $O_{\vec{z}}$ allows all $dep(\vec{y}, x)$, with $\vec{y}$ included in $v\vec{z}$:

$$O_{z_1} \dots O_{z_n} \exists_s x(\phi \wedge dep(\vec{y}, x))$$

4. **Marked indefinites** trigger the obligatory activation of particular dependence or variation atoms.

Final Proposal

$$O_{z_1} \dots O_{z_n} \exists_s x(\phi \wedge \dots)$$

Plain: $dep(\vec{y}, x)$, where $\vec{y} \subseteq v\vec{z}$

SK: $dep(\vec{y}, x)$ with $\vec{y} = \varnothing$

Specific: $dep(\vec{y}, x)$ with $\vec{y} \subseteq \{v\}$

Epistemic: $dep(\vec{y}, x) \wedge var(\vec{z}, x)$ with $\vec{z} = \varnothing$

Non-specific: $dep(\vec{y}, x) \wedge var(\vec{z}, x)$ with $\vec{z} = v$

SU: $dep(\vec{y}, x) \wedge var(\vec{z}, x)$ with $\vec{y} = v$ and $\vec{z} = \varnothing$

Application VI: Interaction with Scope

$\forall z \forall y \exists_s x \phi$

	WS-K $dep(\emptyset, x)$	WS-U $dep(v, x)$	IS $dep(vy, x)$	NS $dep(vyz, x)$
unmarked	✓	✓	✓	✓
specific $dep(\subseteq v, x)$	✓	✓	×	×
non-specific $var(v, x)$	×	×	✓	✓
epistemic $var(\emptyset, x)$	×	✓	✓	✓
specific known $dep(\emptyset, x)$	✓	×	×	×
specific unknown $dep(v, x) \wedge var(\emptyset, x)$	×	✓	×	×

Outline

Negation and Implication

Negation so far was defined for classical first-order literals, assuming negation normal forms.

To express negation in general we can adopt an intensional notion, along the lines of Brasoveanu and Farkas (2011).

(16) **Intensional Negation**

$$\neg\phi(v) \Leftrightarrow \forall w(\phi[v/w] \rightarrow v \neq w)$$

(17) **Semantic Clause for Implication**

$M, X \models \phi \rightarrow_{\exists} \psi \Leftrightarrow$ for some $X' \subseteq X$ s.t. $M, X' \models \phi$ and X' is maximal (i.e. for all X'' s.t. $X' \subset X'' \subseteq X$, it holds $M, X'' \not\models \phi$), we have $M, X' \models \psi$

Definability with weak maximal implication

Let $\vec{r}$ list all variables in scope, and let $\vec{r}'$, $\vec{p}$, and $\vec{q}$ be fresh copies of $\vec{r}$. We write $\vec{z}'$ for the coordinates in $\vec{r}'$ corresponding to $\vec{z}$, and similarly $\vec{z}^p$ and for the corresponding coordinates in $\vec{p}$.

- dep $\leq \subseteq + \to_\exists$, over all teams:

$$\mathrm{dep}(\vec{z}, \vec{u}) \equiv \forall \vec{r}'(\vec{r}' \subseteq \vec{r} \to_\exists (\vec{z} \neq \vec{z}' \vee \vec{u} = \vec{u}'))$$

- $\subseteq \leq$ dep$+ \to_\exists$, over all teams:

$$\vec{x} \subseteq \vec{y} \equiv \exists_{\mathrm{lax}} \vec{r}'((\mathrm{dep}(\vec{r}, \vec{r}') \to_\exists \vec{x} = \vec{y}') \wedge (\mathrm{dep}(\vec{r}', \vec{r}) \to_\exists \vec{r}' = \vec{r}))$$

- var $\leq$ dep$+ \to_\exists$, over nonempty teams:

$$\mathrm{var}(\vec{z}, \vec{u}) \equiv \exists \vec{p} \exists \vec{q}(\mathrm{dep}(\varnothing, \vec{p}) \wedge \mathrm{dep}(\varnothing, \vec{q}) \wedge \vec{p} \subseteq \vec{r} \wedge \vec{q} \subseteq \vec{r}$$
$$\wedge\, \vec{z}^p = \vec{z}^q \wedge \vec{u}^p \neq \vec{u}^q)$$

  where the inclusion atoms are replaced by the preceding definition.

- var $\leq \subseteq + \to_\exists$, over nonempty teams: use the same formula, replacing dep by the first definition above.

Degano 2024, Galliani 2015

Intensional negation: classical case

Assume ϕ is classical. Then:

$$M, T \models \neg_I \phi(v) \quad \Longleftrightarrow \quad \forall i \in T, M, \{i\} \not\models \phi(v)$$

After $\forall w$, the team is $T[M/w] = \{i[a/w] : i \in T,\ a \in M\}$.

Since $\phi[v/w]$ is flat, its maximal satisfying subteam is unique:

$$U_\phi = \{i[a/w] \in T[M/w] : M, \{i[a/w]\} \models \phi[v/w]\}$$

($\Rightarrow$) If some $i \in T$ satisfies $\phi(v)$, then $i[i(v)/w] \in U_\phi$. But on this assignment, $v = w$, contradicting $U_\phi \models v \neq w$.

($\Leftarrow$) If no $i \in T$ satisfies $\phi(v)$, then whenever $i[a/w] \in U_\phi$, we must have $a \neq i(v)$. Hence $U_\phi \models v \neq w$, so $T \models \neg_I \phi(v)$.

Intensional negation of dep and var

- Assume $v \notin \vec{z}\vec{u}$
- For dependence: $\neg_I \mathrm{dep}(\vec{z}, \vec{u}) = \forall w(\mathrm{dep}(\vec{z}, \vec{u}) \rightarrow_\exists v \neq w)$
- Every nonempty maximal dependence-supporting subteam contains all w-extensions, including assignments with $w = v$. Hence
- $M, T \models \neg_I \mathrm{dep}(\vec{z}, \vec{u}) \iff T = \emptyset$
- For variation: if a variation-supporting subteam exists, every maximal one contains an extension with $w = v$. If none exists, the weak maximal implication is false. Thus
- $\neg_I \mathrm{var}(\vec{z}, \vec{u}) \equiv \bot_{\mathrm{abs}}$
- $\neg_I \neg_I \mathrm{dep}(\vec{z}, \vec{u}) \equiv \top$
- $\neg_I \neg_I \mathrm{var}(\vec{z}, \vec{u}) \equiv \bot_{\mathrm{abs}}$

Negation and Epistemic Indefinites

Desideratum: EIs under negation display an NPI behaviour (e.g., *any*).

EIs under negation as in (18) are supported when the initial team contains just $\{w_\varnothing\}$. (In $w_\varnothing$ John read no book, in w_a John read only book a, and so on.)

(18) John does not have *irgend*-book.

a. $\forall w(\exists_s x(\phi(x,w) \wedge dep(vw,x) \wedge var(\varnothing,x)) \rightarrow \mathrm{v} \neq \mathrm{w})$

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

(a) Supporting Team

v	w	x
w_a	$w_\varnothing$	b
w_a	w_a	a
w_a	w_b	b
w_a	w_{ab}	a

(b) Non-Supporting Team

[maximal teams of antecedent in blue]

Negation and Specific Indefinites

- Does the *some*/*all* distinction matters in the semantic clause for maximal implication?
- For union-closed formulas, it does not. The difference is trivialized.
- But not all formulas in our language are union-closed!
- Let's consider what happens in the case of specific (known) indefinites.

(19) John does not have some-SK book.

a. $\forall w(\exists_s x(\phi(x, w) \wedge dep(\varnothing, x)) \rightarrow v \neq w)$

As in (19), specific indefinites under negation are supported by $\{w_\varnothing\}$ (John has no book), and also by $\{w_a\}$ (John has book a and not b) or $\{w_b\}$. But not by $\{w_{ab}\}$ (John has both books).

Supporting and Non-Supporting Teams

(20) John does not have some-SK book.

a. $\forall w(\exists_s x(\phi(x, w) \wedge dep(\varnothing, x)) \rightarrow v \neq w)$

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	a
$w_\varnothing$	w_{ab}	a

(a) Supporting Team

v	w	x
w_a	$w_\varnothing$	a
w_a	w_a	a
w_a	w_b	a
w_a	w_{ab}	a

(b) Non-Supporting Team

v	w	x
w_{ab}	$w_\varnothing$	a
w_{ab}	w_a	a
w_{ab}	w_b	a
w_{ab}	w_{ab}	a

(c) Non-Supporting Team

v	w	x
$w_\varnothing$	$w_\varnothing$	b
$w_\varnothing$	w_a	b
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

(d) Supporting Team

v	w	x
w_a	$w_\varnothing$	b
w_a	w_a	b
w_a	w_b	b
w_a	w_{ab}	b

(e) Supporting Team

v	w	x
w_{ab}	$w_\varnothing$	b
w_{ab}	w_a	b
w_{ab}	w_b	b
w_{ab}	w_{ab}	b

(f) Non-Supporting Team

Outline

Indefinites and Disjunction

Do languages also exhibit a variety of disjunctions? How can such diversity be modelled?

ATOM	DISJ.	CLASS	INDEFINITE	DISJUNCTION
$dep(\varnothing, x)$	$\mathbin{\vee\!\!/}$	specific known	Lithuanian *kai*	Finnish *vai*
$dep(v, x)$	$\mathbin{\vee\!\!/}_v$	specific	Ossetic *-dær*	Diwa *khí*
$var(\varnothing, x)$	$\vee_s^{\mathbf{var}}$	epistemic	German *irgend-*	Russian *to li ... to li*
$var(v, x)$	$\vee_s^{v\text{-}\mathbf{var}}$	non-specific	Russian *-nibud'*	Diwa *ba*

Variety of Indefinites and Disjunction Across Languages.

Epistemic disjunction

(21) a. John Gita–*hari* Mala–*hari* hamuuna.
'John met Gita or Mala. In fact, I know who.'

b. John kawa–*hari* hamuuna.
'John met somebody. In fact, I know who.'

(22) a. John Gita–*hari* Mala–*hari* hamuuna.
'John met Gita or Mala. In fact, I know who.'

b. John Gita–*də* Mala–*də* hamuuna.
'John met Gita or Mala. #In fact, I know who.'

Similar epistemic disjunctions: Russian *to li . . . to li* (Ivlieva 2024). Possibly Turkish *ya da* (Mekik & Singh 2016).

Non-specific disjunction

Dawson (2020): Tiwa *ba* forms a disjunction that is obligatorily narrow scope.

(23) a. Saldi **ba** Mukton khâl lí-ga.
'Saldi or Mukton ran away.'

Under a distributive quantifier, *ba* cannot take wide scope:

(24) Sógol loró–râw Lastoi *ba* Mukton–go ni–na lí-ga.
every priest–PL Lastoi BA Mukton–ACC see–INF go–PFV
'Every priest went to see Lastoi or Mukton.'

$\forall > \vee$	✓	possibly different alternatives for different priests
$\vee > \forall$	$\times$	one fixed alternative for all priests

Specific disjunction

Dawson (2020): Tiwa *khí* is the mirror image of *ba*: it is obligatorily wide scope.

(25) Sógol loró–râw Lastoi *khí* Mukton–go ni–na lí-ga.
every priest–PL Lastoi KHI Mukton–ACC see–INF go–PFV
'Every priest went to visit Lastoi or Mukton.'

$\forall > \vee$	$\times$	possibly different alternatives for different priests
$\vee > \forall$	✓	one fixed alternative for all priests

Specific-known disjunction?

Languages can lexicalize specific-known indefinites: the speaker has a particular witness in mind, but withholds its identity.

But there seem to be no clear cases of specific-known disjunctions.

Why?

- Specific-known indefinites make the identity available but irrelevant.
- Disjunctions make the alternatives explicit and salient.

Interrogative disjunction

Could alternative questions be specific-known disjunctions?

(26) Haluatko kahvia vai teetä?
want.2SG.Q coffee.PART or tea.PART
'Do you want coffee or tea?'

The question asks the addressee to resolve the alternatives in a specific way.

Variety of Disjunctions

$M, T \models \phi \vee \psi \Leftrightarrow T = T_1 \cup T_2$ for teams T_1 and T_2 s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

Claim: the variety of disjunctions can be captured by the interplay of two basic constraints.

Non-empty: $T_1 \neq \varnothing$ and $T_2 \neq \varnothing$

Uniqueness: $T_1 = \varnothing$ or $T_2 = \varnothing$

(The non-empty constraint is related, but not equivalent, to the neglect-zero approach of Aloni (2022). The connection will be made explicit later.)

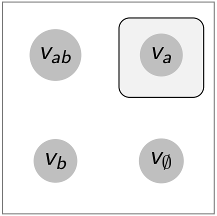
Non-empty

$M, T \models \phi \vee \psi \Leftrightarrow T = T_1 \cup T_2$ for teams T_1 and T_2 s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

$+$

Non-empty: $T_1 \neq \varnothing$ and $T_2 \neq \varnothing$

We indicate this constraint on disjunction with $\phi \vee_{+\text{ne}} \psi$.



$M, T \models Fa$
$M, T \not\models Fa \vee_{+\text{ne}} Pb$

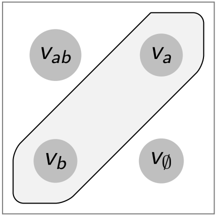
Uniqueness

$M, T \models \phi \vee \psi \Leftrightarrow T = T_1 \cup T_2$ for teams T_1 and T_2 s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

$+$

Uniqueness: $T_1 = \varnothing$ or $T_2 = \varnothing$

We indicate this constraint on disjunction with $\phi \vee_{+\mathrm{u}} \psi$.



$M, T \not\models Pa$
$M, T \not\models Pa \vee_{+\mathrm{u}} Pb$

(The addition of this constraint results in the intuitionistic disjunction.)

The second sort

Since our framework is two-sorted, it is useful to generalize disjunction to the variable for the actual world v.

v-**Uniqueness:**

$M, T \models \phi \vee^{v}_{+\mathrm{u}} \psi \Leftrightarrow$

1. $M, T \models \phi \vee \psi$
2. for all $w \in T(v)$, $M, T_{v=w} \models \phi \vee_{+\mathrm{u}} \psi$

T	v	...
i_1	v_1	...
i_2	v_1	...
i_3	v_2	...
i_4	v_2	...

(Further generalization: the scope of disjunction can effectively be captured by evaluating the disjunction in the team subsetted by the relevant variables on which it covaries.)

The second sort

v-**Non-empty:**

$M, T \models \phi \vee^{v}_{+\text{ne}} \psi \Leftrightarrow$

1. $M, T \models \phi \vee \psi$
2. for some $w \in T(v)$, $M, T_{v=w} \models \phi \vee_{+\text{ne}} \psi$

T	v	...
i_1	v_1	...
i_2	v_1	...
i_3	v_2	...
i_4	v_2	...

Epistemic Disjunction

Epistemic Disjunction will be captured by **Non-empty**: $\phi \vee_{+\text{ne}} \psi$

T	v
i_1	$\boxed{v_1}$
i_2	$\boxed{v_2}$

In episodic contexts, non-empty leads to the required ignorance inference.

In a two-sorted system, we can treat modals as quantifiers over worlds.

Epistemic modals will range over the possible values for the actual world, encoded by v.

(27) a. John likes Mary or Sue.

b. $\rightsquigarrow \Diamond_e P(j, m, v) \wedge \Diamond_e P(j, s, v)$

Epistemic Disjunction

Under universal quantifiers:

Non-empty gives us the co-variation/distributive reading in (a).

Non-empty and taking the disjunction to have wide-scope with respect to v (v-**Uniqueness**), gives us the ignorance reading.

(28) Everyone likes Sue or$_{epi}$ Mary.

a. Everyone likes Sue or Mary. (Someone likes Sue and Someone likes Mary.)

b. Everyone likes Sue, or everyone likes Mary. The speaker does not know which one.

Specific Disjunction

Specific disjunction is captured by v-**Uniqueness**: $\phi \vee^{v}_{+\mathbf{u}} \psi$

(29) Everyone$_x$ likes Sue or$_{sp}$ Mary.
$\rightsquigarrow$ Everyone likes Sue, or everyone likes Mary. (Or $>$ Every)

T	v	x
i_1	v_1	d_1
i_2	v_1	d_2
i_3	v_2	d_1
i_4	v_2	d_2

Non-Specific Disjunction

Non-specific disjunction is captured by v-**Non-empty**: $\phi \vee^{v}_{+\text{ne}} \psi$

(30) Everyone$_x$ likes Sue or$_{nsp}$ Mary.
$\rightsquigarrow$ Everyone likes Sue or Mary (Every $>$ Or).

T	v	x
i_1	v_1	d_1
i_2	v_1	d_2
i_3	v_2	d_1
i_4	v_2	d_2

Negation

Dealing with negation comes with several design choices.

$$\neg_I \phi(v) \Leftrightarrow \forall w(\phi[v/w] \rightarrow v \neq w)$$

Does it work for the current treatment of disjunctions?

Negation

$$\neg_I \phi(v) \Leftrightarrow \forall w(\phi[v/w] \rightarrow v \neq w)$$

Implication: some maximal subset of the team supporting the antecedent supports the consequent.

(31) John doesn't like Mary or Sue.

a. $\neg_I(P(m, v) \vee P(s, v))$

b. $\forall w((P(m, w) \vee P(s, w)) \rightarrow v \neq w)$

T	v	w
i_1	$v_\varnothing$	v_{ms}
i_2	$v_\varnothing$	v_m
i_3	$v_\varnothing$	v_s
i_4	$v_\varnothing$	$v_\varnothing$

Note: $\neg_I(P(m, v) \vee P(s, v)) \equiv \neg_I P(m, v) \wedge \neg_I P(s, v)$

Negation and predictions (specific)

Dawson (2020): specific disjunctions under negation give rise to wide-scope reading.

(32) John doesn't like Mary or_{sp} Sue.

a. $\neg_I(P(m,v) \vee^{v}_{+\mathrm{u}} P(s,v))$

b. $\forall w(P(m,v) \vee^{v}_{+\mathrm{u}} P(s,v)) \rightarrow v \neq w)$

T	v	w
i_1	v_s	$\boxed{v_{ms}}$
i_2	v_s	$\boxed{v_s}$
i_3	v_s	v_s
i_4	v_s	$v_\varnothing$

Note: $\neg_I(P(m,v) \vee^{v}_{+\mathrm{u}} P(s,v)) \equiv \neg_I P(m,v) \vee \neg_I P(s,v)$

Negation and predictions (non-specific)

Dawson (2020): non-specific disjunctions under negation give rise to narrow-scope reading.

(33) John doesn't like Mary or$_{nsp}$ Sue.

a. $\neg_I(P(m,v) \vee^{v}_{+\text{ne}} P(s,v))$

b. $\forall w((P(m,v) \vee^{v}_{+\text{ne}} P(s,v)) \rightarrow v \neq w)$

T	v	w
i_1	$v_\varnothing$	$\boxed{v_{ms}}$
i_2	$v_\varnothing$	$\boxed{v_m}$
i_3	$v_\varnothing$	$\boxed{v_s}$
i_4	$v_\varnothing$	$v_\varnothing$

Note:
$\neg_I(P(m,v) \vee^{v}_{+\text{ne}} P(s,v)) \equiv \neg_I(P(m,v) \vee P(s,v)) \equiv \neg_I P(m,v) \wedge \neg_I P(s,v)$

Negation and predictions (epistemic)

Prediction: epistemic disjunctions are ambiguous.

(34) John doesn't like Mary or Sue.

a. John doesn't like Mary, or John doesn't like Sue. The speaker does not know who.

b. John doesn't like Mary or Sue.

The empirical picture is different.

Epistemic disjunctions under negation (Ivlieva 2024) seem to allow only for the reading in (a).

A puzzle

Epistemic indefinites under sentential negation display a NPI behaviour.

Epistemic disjunctions under sentential negation display a PPI behaviour.

Free Choice Disjunction

- Many languages have free choice indefinites.
- But there appears to be no free choice disjunction.
- Conjecture: lack of domain widening effect in disjunction.
- But how to characterize a free choice disjunction?

Free Choice

Free Choice

$M, T \models \phi \vee_{+\text{fc}} \psi \Leftrightarrow$ for all $w \in T(v)$, $M, T_{v=w} \models \phi \vee_{+\text{ne}} \psi$

T	v	...
i_1	v_1	...
i_2	v_1	...
i_3	v_2	...
i_4	v_2	...

Some facts

Under a (possibility) modal, we get the free choice reading, similarly to Aloni (2022) neglect-zero approach $(\cdot)^+$:

(35) a. $\Diamond(\phi \vee \psi)^+ \models \Diamond\phi \wedge \Diamond\psi$

b. $\Diamond(\phi \vee_{\text{+fc}} \psi) \models \Diamond\phi \wedge \Diamond\psi$

(36) a. $\neg\Diamond(\phi \vee \psi)^+ \models \neg\Diamond\phi \wedge \neg\Diamond\psi$

b. $\neg\Diamond(\phi \vee_{\text{+fc}} \psi) \models \neg\Diamond\phi \wedge \neg\Diamond\psi$

A notable difference:

(37) a. $(\phi \vee \psi)^+ \models \Diamond_e\phi \wedge \Diamond_e\psi$

b. $\phi \vee_{\text{+fc}} \psi \models \phi \wedge \psi$

Disjunction as a Choice Variable

Introduce a fresh choice variable $b \in \{L, R\}$

The values of b identify which disjunct is selected.

$$M, T \models \phi \vee_{\Gamma} \psi$$

iff there is a b-extension $\widehat{T}$ of T such that

1. $M, \widehat{T}_{b=L} \models \phi$
2. $M, \widehat{T}_{b=R} \models \psi$
3. $M, \widehat{T} \models \Gamma(b)$

Different disjunctions correspond to different constraints $\Gamma(b)$ on b.

The Relational View

Equivalently, the b-extension determines a relation $C \subseteq T \times \{L, R\}$, where

$$i \, C \, j \quad \Longleftrightarrow \quad i[j/b] \in \widehat{T}$$

Constraint on C	**Constraint on b**	**Disjunction**
C constant	$\mathrm{dep}(\varnothing, b)$	inquisitive
C functional	$\mathrm{dep}(\mathrm{Dom}(T), b)$	strict
C total	no dependence requirement	lax
C determined by v	$\mathrm{dep}(v, b)$	specific / v-unique
C varies within a v-cell	$\mathrm{var}(v, b)$	non-specific / distributive

Expressive power

- At the level of sentences: $\mathcal{L}_{\textsf{dep,var}} \equiv \mathrm{ESO}$
- ESO is already obtained from dependence atoms alone. Adding var does not increase sentence-level expressive power: $\mathsf{var}(\vec{z}, u)$ is first-order definable over the team relation.
- At the level of formulas: $\mathcal{L}_{\textsf{dep,var}} \;<\; \mathrm{ESO}(R)$
- Here, $\mathrm{ESO}(R)$ is existential second-order logic over the expanded vocabulary containing a distinguished relation symbol R that encodes the team:

$$R(\vec{a}) \quad\Longleftrightarrow\quad \vec{a} = i(\vec{x}) \text{ for some } i \in T$$

  where $\vec{x}$ lists the free variables of the formula.
- Totality is $\forall u\, R(u)$ is ESO-definable but not definable in $\mathcal{L}_{\textsf{dep,var}}$.

Väänänen 2007, Kontinen & Väänänen 2009, Galliani 2015

Expressive power

- At the level of sentences: $\mathcal{L}_{\mathsf{dep},\mathsf{var},\to_\exists} \equiv \mathrm{SO}$
- Upper bound: maximal implication is second-order definable, since maximality quantifies over subteams.
- Lower bound: $\to_\exists$ yields a form of contradictory negation on canonical teams:

$$M, T \models \varphi \to_\exists \bot \iff \text{no nonempty } S \subseteq T \text{ satisfies } \varphi$$

- Together with definable totality, this simulates universal second-order quantification: $\forall F\, \psi \equiv \neg\exists F\, \neg\psi$
- Hence one can adapt the standard function-normal-form translation for SO along the lines of Yang (2013)'s proof for intuitionistic dependence logic.
- At the level of formulas, over all teams: $\mathcal{L}_{\mathsf{dep},\mathsf{var},\to_\exists} < \mathrm{SO}(R)$
- Strictness comes from the empty-team: formulas cannot inspect the model from the empty input team, while $\mathrm{SO}(R)$ can.

Exercise 1

Consider the ambiguous sentence:

(1) *Ali must hire a philosopher.*

(i) Identify the two scope readings of (1) and translate each:

  (a) in classical quantified modal logic
  (b) using Degano and Aloni's dependence atoms. Here translate the modal as quantification over possible worlds.

(ii) Now consider a marked indefinite determiner IND_x that activates the variation atom $\mathrm{var}(v, x)$:

  (2) *Ali must$_w$ hire* IND_x *philosopher.*

  Which reading of (1) does Degano and Aloni's analysis predict for (2)? Explain the role of $\mathrm{var}(v, x)$.

Exercise 2

Let $W = \{w\}$ and $D = \{a, b\}$. Let $T_0 = \{i_0\}$ with $i_0(v) = w$, and let $T_1 = T_0[D/y]$, where x, y are fresh individual variables.

1. Decide all four claims below. For every positive answer, exhibit a supplement. For every negative answer, prove impossibility.

$$T_i \models \exists_{\mathrm{s}} x \, \mathrm{var}(v, x) \qquad T_i \models \exists_{\mathrm{l}} x \, \mathrm{var}(v, x) \quad (i = 0, 1)$$

2. Generalize the calculation. Assuming x is fresh, $Z \subseteq \mathrm{dom}(T)$, and $|D_x| \geq 2$, prove

$$T \models \exists_{\mathrm{s}} x \, \mathrm{var}(Z, x) \quad \Longleftrightarrow \quad \exists i \neq j \in T i(Z) = j(Z)$$

3. Put $\psi := \exists_{\mathrm{s}} x \, \mathrm{var}(\varnothing, x)$. Construct a team T whose only column is an irrelevant y such that $T \models \psi$ but $T{\restriction}_{\varnothing} \not\models \psi$. Identify the step in the usual locality proof that fails.

Exercise 3

Consider the following team-based possibility operator:

$M, T \models \blacklozenge\phi \iff \exists S(\varnothing \neq S \subseteq T \text{ and } M, S \models \phi)$

1. Let α be a flat first-order formula and T nonempty. Show that:
   $M, T \models \blacklozenge\alpha \iff M, \{i\} \models \alpha$ for some $i \in T$
2. Determine the support set of $\blacklozenge\mathrm{dep}(\vec{z}, x)$ and $\blacklozenge\mathrm{var}(\vec{z}, x)$.
3. Discuss
   - why the black diamond is a natural analysis of epistemic *might*.
   - why it cannot replace modal quantification in the present theory, which is modelled as existential lax quantification with inclusion atoms.
   - what each of the two analyses allows us to model.